CHAPTER 9

THEORIES OF ELASTIC FAILURE

10.1. Theories of Failure Under Static Load

Mechanical components are subjected to several types of loads simultaneously. For example , a power screw is subjected to torsional moment as well as axial force. Similarly, an overhang crank is subjected to combined bending and torsional moment. The bolts of the bracket are subjected to forces that cause tensile stress and shear stress. Crankshaft, propeller shaft, and connecting rod are examples of the components subjected to complex loads. When the component is subjected to several types of loads, combined stresses are induced. For example, torsional moment induces torsional shear stress, while bending moment causes bending stress in the transmission shaft. The failures of such components are broadly classified into two groups; i.e. Elastic failure, yielding and fracture. Elastic failure results in excessive elastic deformation, which makes the machine component unfit to perform its function satisfactorily. Yielding results in excessive plastic deformation after the yield point stress is reached, while fracture results in breaking the component into two or more pieces. Theories of failure discussed in this article are applicable to elastic failure of machine parts.

The design of machine parts subjected to combined loads should be related, to experimentally determined properties of material under 'similar' conditions. However, it is not possible to conduct such tests for different possible combinations of loads and obtain mechanical properties. In practice, the mechanical properties are obtained from simple tension test. They include yield strength, ultimate tensile strength and percentage elongation. In tension test, the specimen is axially loaded in tension. It is ,not subjected to either bending moment or torsional moment or a combination of loads. Theories of elastic failure provide a relationship between the strength of machine component subjected to complex state of stresses with the mechanical properties obtained in tension test. With the help of these theories, the data obtained in tension test can be used to determine the dimensions of the component, irrespective of the nature of stresses induced in the component due to complex Several theories have been proposed, each assuming a different hypothesis of failure. The principal theories of elastic failure are as follows:

(i) Maximum principal stress theory (Rankine's theory);
(ii) Maximum shear stress theory (Coulomb, Tresca and Guest's theory);

(iii) Distortion energy theory (Huber von Mises and Hencky's theory);

(iv) Maximum strain theory (St. Venant's theory); and

(v) Maximum total strain energy theory (Haigh's theory).

We will discuss the first three theories in this chapter. Let us assume that σ_1, σ_2 and σ_3 as the principal stresses induced at a point on the machine part as a result of several types of loads. We will apply the theories of failure to obtain relationship between σ_1, σ_2 and σ_3 on one hand and the properties of material such as yield strength in tension (S_{yt}) or ultimate strength in tension (S_{ut}) on the other.

i. Maximum principal (Normal) stress Theory

The criterion of failure is accredited to the British engineer, W.J.M. Rankine (1850). The theory states that the failure of the mechanical component, subjected to bi-axial or tri-axial stresses, occurs when the maximum principal stress reaches the yield or ultimate strength of the material.

If σ_1, σ_2 and σ_3 are the three principal stresses at point on the component and

$\sigma_1 > \sigma_2 > \sigma_3$

Then according to this theory, the failure occurs whenever,

$\sigma_1 = S_{yt}$ or $\sigma_1 = Sut$ …. (1)

Whichever is applicable.

The theory considers only the maximum of principal stresses and disregards the influence of the other principal stresses. The dimensions of the component are determined by using a factor of safety.

For tensile stresses,

$$\sigma_1 = \frac{S_{yt}}{(f_s)} \text{ or } \quad \sigma_1 = \frac{S_{ut}}{(f_s)} \quad \ldots. \quad (2)$$

For compressive stresses,

$\sigma_1 = \frac{S_{yc}}{(f_s)}$ or $\sigma_1 = \frac{S_{uc}}{(f_s)}$ ...(3)

Where (f_s) = Factor of safety

Region of Safety: The construction of a region of safety for bi-axial stresses is illustrated in Fig. 1. The two principal stresses σ_1 and σ_2 are plotted on X and Y-axes respectively. Tensile stresses are considered as positive, while compressive stresses as negative. It is further assumed that

$$S_{yc} = S_{yt}$$

It should be noted that,

(i) The equation of vertical line to the positive side of X-axis is (x = + a)

(ii) The equation of vertical line to the negative side of X-axis is (x = - a)

(iii) The equation of horizontal line to the positive side of Y-axis is (y = + b)

(iv) The equation of horizontal line to the negative side of Y-axis is (y = - b)

The borderline for region of safety for this theory can be constructed in the following way:

Step 1: Suppose $\sigma_1 > \sigma_2$. As per this theory we will consider only the maximum of principal stresses (σ_1) and disregard the other principal stress (σ_2).

Suppose (σ_1) is tensile stress. The limiting value of (σ_1) is yield stress (S_{yt}). Therefore, the boundary line will be,

$\sigma_1 = +S_{yt}$

A vertical line AB is constructed such that $\sigma_1 = + S_{yt}$.

Step 2: Suppose $\sigma_1 > \sigma_2$ and (σ_1) is compressive stress. The limiting value of (σ_1) is compressive yield stress ($-S_{yc}$). Therefore, the boundary line will be,

$\sigma_1 = -S_{yc}$

A vertical line DC is constructed such that $\sigma_1 = -S_{yc}$.

Step 3: Suppose $\sigma_2 > \sigma_1$**.** As per this theory we will consider only the maximum of principal stresses (σ_2) and disregard the other principal stresses (σ_1).

Suppose (σ_2) is tensile stress. The limiting value of (σ_2) is yield stress (S_{yt}). Therefore, the boundary line will be,

σ_2 = + S_{yt}

A horizontal line CB is constructed such that

σ_2 = + $s_{yt.}$

Step 4: Suppose $\sigma_2 > \sigma_1$, and (σ_2) is compressive stress. The limiting value of (σ_2) is compressive yield stress (-S_{yc}). Therefore, the boundary line will be,

σ_2 = -S_{yc}

A horizontal line DA is constructed such that

σ_2 = - S_{yc}.

The complete region of safety is area ABCD. Since, we have assumed (S_{yc} = S_{yt}), ABCD is square.

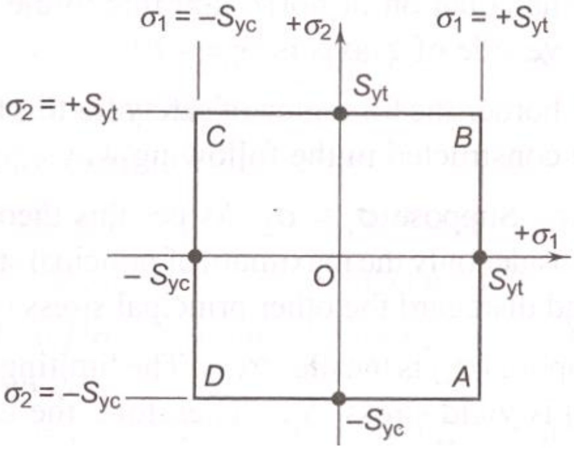


Fig.1 Boundary for maximum principal stress theory under bi-axial stress

According to the maximum principal theory of failure, if a point with co-ordinates (σ_1,σ_2) falls outside this square, then it indicates failure condition. On the other hand, if the point falls inside the square, the design is safe and failure may not occur.

Experimental investigations suggest that maximum principal stress theory gives good predictions for brittle materials. However, it is not recommended for ductile materials.

ii. Maximum shear stress theory

This criterion of failure is accredited to C.A. Coulomb, H. Tresca and J.J. Guest. The theory states that the failure of a mechanical component subjected to bi-axial or tri -axial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the tension test, when yielding

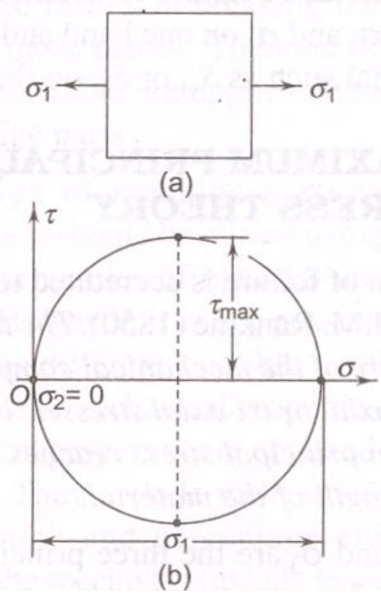


Fig.2 a) Stress in simple tension test
b) Mohr's circle for stress

starts. In tension test, the specimen is subjected to uni-axial stress (σ_1) and (σ_2 = 0). The stress in the specimen of tension test and the corresponding Mohr's circle diagram are shown in Fig. 2. From the figure,

$$\tau_{max} = \frac{\sigma_1}{2}$$

When the specimen starts yielding (σ_1 = S_{yt}), the above equation is written as

$$\tau_{max} = \frac{S_{yt}}{2}$$

Therefore, the maximum shear stress theory predicts that the yield strength in shear is half of the yield strength in tension, i.e.,

$$S_{sy} = 0.5\ S_{yt} \qquad \text{....(4)}$$

If σ_1, σ_2 and σ_3 are the three principal stresses at a point on the component, the shear stresses on three different planes are given by,

$$\tau_{12} = \left(\frac{\sigma_1 - \sigma_2}{2}\right), \quad \tau_{23} = \left(\frac{\sigma_2 - \sigma_3}{2}\right), \tau_{31} = \left(\frac{\sigma_3 - \sigma_1}{2}\right) \qquad \text{....(a)}$$

The largest of these stresses is equated to (τ_{max}) or ($\frac{S_{yt}}{2}$). Considering factor of safety,

$$\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{S_{yt}}{2(fs)} \text{ or } \sigma_1 - \sigma_2 = \frac{S_{yt}}{(fs)}, \ \sigma_2 - \sigma_3 = \frac{S_{yt}}{(fs)}, \quad \sigma_3 - \sigma_1 = \frac{S_{yt}}{(fs)} \qquad \text{...(5)}$$

The above relationships are used to determine the dimensions of the component.
Refer to expression (a) again and equating the largest shear stress (τ_{max}) to ($S_{yt}/2$),

$$\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{S_{yt}}{2} \text{ or } \sigma_1 - \sigma_2 = S_{yt} \qquad \text{...(b)}$$

Similarly,

$\sigma_2 - \sigma_3 = S_{yt}$ ...(c) , $\sigma_3 - \sigma_1 = S_{yt}$ ...(d)

For compressive stress,

$\sigma_1 - \sigma_2 = -S_{yc}$ ....(e) , $\sigma_2 - \sigma_3 = -S_{yc}$ ...(f) , $\sigma_3 - \sigma_1 = -S_{yc}$ ....(g)

The above equations can be written as,

$\sigma_1 - \sigma_2 = \pm S_{yt}$ (Assuming that $S_{yc} = S_{yt}$)

$\sigma_2 - \sigma_3 = \pm S_{yt}$, $\sigma_3 - \sigma_1 = \pm S_{yt}$

Region of safety: For bi-axial stresses, $\sigma_3 = 0$

The above equations can be written as,

$\sigma_1 - \sigma_2 = \pm S_{yt}$ ...(h) , $\sigma_2 = \pm S_{yt}$ ...(i) , $\sigma_1 = \pm S_{yt}$ ...(j)

It will be observed at a later stage that equations (h) are applicable in second and fourth quadrants, while equations (i) and (j) are applicable in first and third quadrants of the diagram.

The construction of the region of safety is illustrated in Fig. 3. The two principal stresses σ_1 and σ_2 are plotted on X and Y axes respectively. Tensile stresses are considered as positive, while compressive stresses as negative.

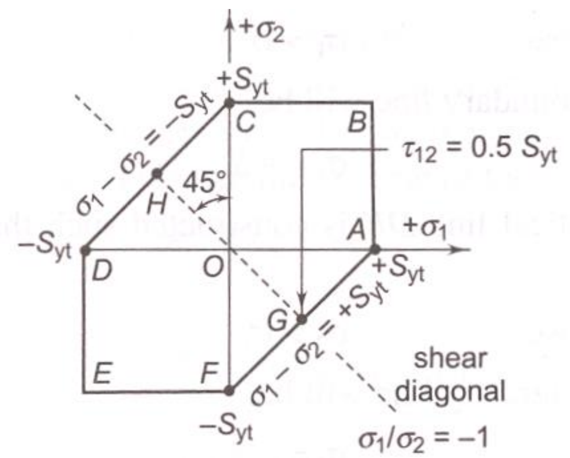


Fig.3 : Boundary for Maximum Shear Stress Theory under bi-axial stresses

It should be noted that,

(i) The equation (x - y = - a) indicates a straight line in second quadrant with (- a) and (+ a) as intercepts on X and Y axes respectively.

(ii) (The equation (x - y = + a) indicates a straight line in fourth quadrant with (+ a) and (- a) as intercepts on X and Y axes respectively.

iii. Distortion - Energy Theory

Distortion-Energy theory was advanced by M.T. Huber in Poland (1904) and independently by R. von Mises in Germany (1913) and H. Hencky (1925). It is known as the Huber von Mises and Hencky's theory. The theory states that the failure of the mechanical component subjected to bi-axial or tri-axial stresses occurs, when the strain energy of distortion per unit volume at any point in the component, becomes equal to the strain energy of distortion per unit volume in a standard specimen of tension test, when yielding starts.

A unit cube subjected to the three principal σ_1, σ_2 and σ_3 is shown in Fig. 4 (a).

The total strain energy U of the cube is given by,

$$U = \frac{1}{2}\sigma_1 \varepsilon_1 + \frac{1}{2}\sigma_2 \varepsilon_2 + \frac{1}{2}\sigma_3 \varepsilon_3 \qquad \text{(a)}$$

ε_1, ε_2 and ε_3 are strains in respective directions.

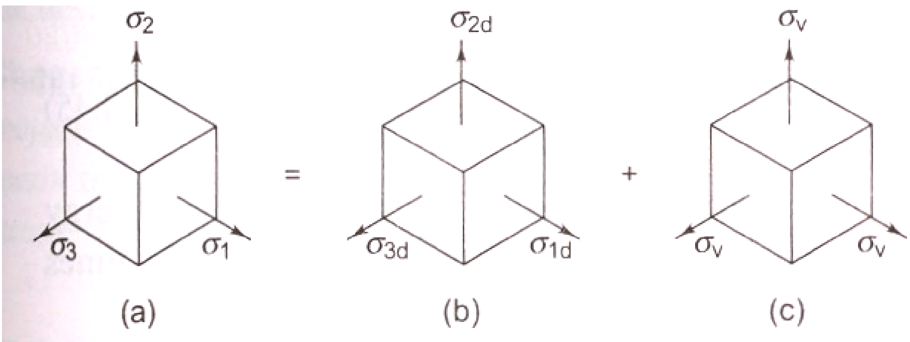


Fig. 4 (a) Element with Tri-axial Stresses (b) Stress Components due to Distortion of Element (c) Stress Components due to Change of Volume.

Also,

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu(\sigma_1 + \sigma_3)\right]$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \mu(\sigma_1 + \sigma_2)\right] \qquad \text{(b)}$$

Substituting the above expressions in Eq. (a),

$$U = \frac{1}{2E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \qquad \text{(c)}$$

The total strain energy U is resolved into two components: one U_v corresponding to the change of volume with no distortion of the element and the other U_d corresponding to the distortion of the element with no change of volume. Therefore,

$$U = U_v + U_d \qquad \text{(d)}$$

The corresponding stresses are also resolved into two components as shown in Figs 4 (b) and (c). From the figure,

$$\begin{aligned} \sigma_1 &= \sigma_{1d} + \sigma_v \\ \sigma_2 &= \sigma_{2d} + \sigma_v \\ \sigma_3 &= \sigma_{3d} + \sigma_v \end{aligned} \qquad \text{(e)}$$

The components σ_{1d}, σ_{2d}, and σ_{3d} cause distortion of the cube, while the component σ_v results in volumetric change. Since the components σ_{1d}, σ_{2d} and σ_{3d} do not change the volume of the cube,

$$\varepsilon_{1d} + \varepsilon_{2d} + \varepsilon_{3d} = 0 \qquad \text{(f)}$$

Also,

$$\begin{aligned} \varepsilon_{1d} &= \frac{1}{E}\left[\sigma_{1d} - \mu(\sigma_{2d} + \sigma_{3d})\right] \\ \varepsilon_{2d} &= \frac{1}{E}\left[\sigma_{2d} - \mu(\sigma_{1d} + \sigma_{3d})\right] \\ \varepsilon_{3d} &= \frac{1}{E}\left[\sigma_{3d} - \mu(\sigma_{1d} + \sigma_{2d})\right] \end{aligned} \qquad \text{(g)}$$

Substituting Eq. (g) in Eq. (f),

$$(1-2\mu)(\sigma_{1d} + \sigma_{2d} + \sigma_{3d}) = 0$$

$$\text{Since} \qquad (1-2\mu) \neq 0$$

$$\therefore \qquad (\sigma_{1d} + \sigma_{2d} + \sigma_{3d}) = 0 \qquad \text{(h)}$$

Substituting Eq. (h) in Eq. (e),

$$\sigma_v = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \qquad \text{(j)}$$

The strain energy U, corresponding to the change of volume for the cube is given by,

$$U_v = 3\left[\frac{\sigma_v \varepsilon_v}{2}\right] \qquad \text{(k)}$$

$$\text{Also} \qquad \varepsilon_v = \frac{1}{E}[\sigma_v - \mu(\sigma_v + \sigma_v)]$$

$$\text{or} \qquad \varepsilon_v = \frac{(1-2\mu)\sigma_v}{E}. \qquad \text{(l)}$$

From expressions (k) and (l),

$$U_v = \frac{3(1-2\mu)\sigma_v^2}{2E} \qquad \text{(m)}$$

Substituting expressions (j) in the above equation,

$$U_v = \frac{(1 - 2\mu)(\sigma_1 + \sigma_2 + \sigma_3)^2}{6E} \qquad \text{(n)}$$

From expressions (c) and (n),

$$U_d = U - U_v$$

or

$$U_d = \left(\frac{1 + \mu}{6E}\right) \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \qquad \text{....(6)}$$

In simple tension test, when the specimen starts yielding,

$$\sigma_1 = S_{yt} \quad \text{and} \quad \sigma_2 = \sigma_3 = 0$$

Therefore, $$U_d = \left(\frac{1 + \mu}{3E}\right) S_{yt}^2 \qquad \text{....(7)}$$

From Eqs (6) and (7), the criterion of failure for the distortion energy theory is expressed as

$$2S_{yt}^2 = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

or

$$S_{yt} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} \qquad \text{....(8)}$$

Considering factor of safety,

$$\frac{S_{yt}}{(fs)} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} \quad \text{....(9)}$$

For bi-axial stresses (σ_3 = 0),

$$\frac{S_{yt}}{(fs)} = \sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)} \quad \text{...(10)}$$

A component subjected to pure shear stress and the corresponding Mohr's circle diagram is shown in Fig. 5.

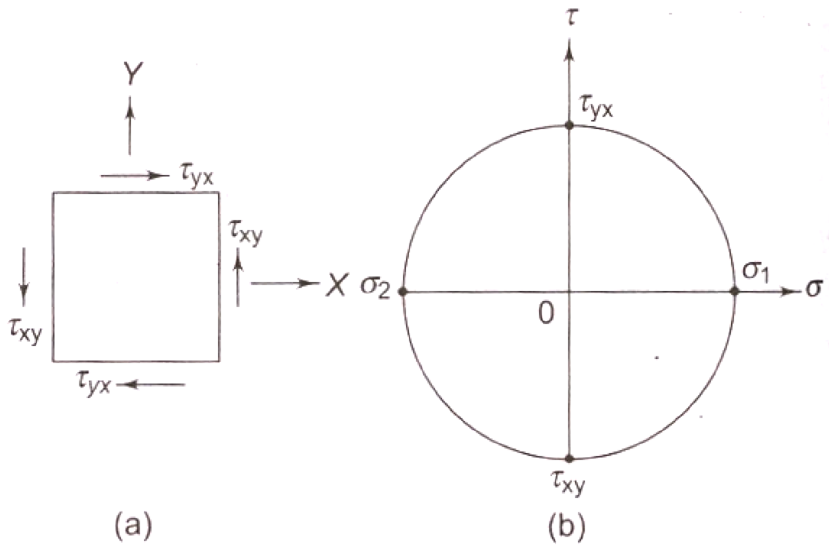


Fig.5 (a) Element subjected to Pure Shear Stresses (b) Mohr's circle for Shear Stresses

From the figure,

$$\sigma_1 = -\sigma_2 = \tau_{xy} \quad \text{and} \quad \sigma_3 = 0$$

Substituting these values in Eq. (8)

$$S_{yt} = \sqrt{3}\,\tau_{xy}$$

Replacing (τ_{xy}) by (S_{sy}),

$$S_{sy} = \frac{S_{yt}}{\sqrt{3}} = 0.577\ S_{yt}$$

Therefore, according to the distortion-energy theory, the yield strength in shear is 0.577 times the yield strength in tension.

Experiments have shown that the distortion-energy theory is in better agreement for predicting the failure of ductile component, than any other theory of failure.

10.2. <u>Selection and use of failure theories</u>

The plots of three theories of failure on σ_1, σ_2 coordinate system are shown in Fig. 6. While selecting theories of failure, following points should be noted:

(i) Ductile materials typically have same tensile strength and compressive strength. Also, yielding is the criterion of failure in ductile materials. In maximum shear stress theory and distortion energy theory, it is assumed that the yield strength in tension (S_{yt}) is equal to yield strength in compression (S_{yc}). Also, the criterion of failure is yielding. Therefore, maximum shear stress theory and distortion energy theory are used for ductile materials.

(ii) Distortion energy theory predicts yielding with precise accuracy in all four quadrants. The design calculations involved in this theory are slightly complicated as compared with those of maximum shear stress theory.

(iii) The hexagonal diagram of maximum shear stress theory is inside the ellipse of distortion energy theory. Therefore, maximum shear stress theory gives results on the conservative side. On the other hand, distortion energy theory is more liberal.

(iv) The graph of maximum principal stress theory is same as that of maximum shear stress theory in first and third quadrants. However, the graph of maximum principal stress theory is outside the ellipse of distortion energy theory in second and fourth quadrants. Thus it would be dangerous to apply maximum principal stress theory in these regions, since it might predict safety, when in fact no safety exists.

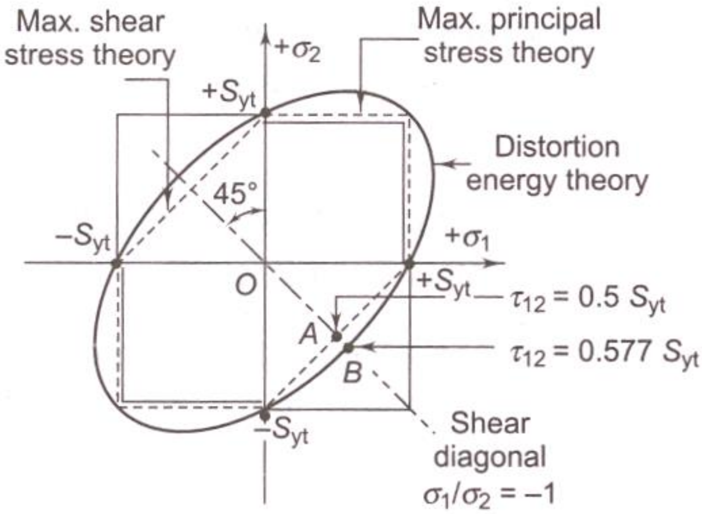


Fig. 6 : Comparison of Theories of Failure

(v) Maximum shear stress theory is used for ductile materials, if dimensions need not be held too close and a generous factor of safety is used. The calculations involved in this theory are easier than those of distortion energy theory.

(vi) Distortion energy theory is used when the factor of safety is to be held in close limits and the cause of failure of component is being investigated. This theory predicts the failure most accurately.

(vii) The compressive strength of brittle material is much higher than their tensile strength. Therefore, the failure criterion should show a difference in tensile and compressive strength. On this account, maximum principal stress theory is used for brittle materials. Also, brittle materials do not yield and they fail by fracture.

To summarize, maximum principal stress theory is a proper choice for brittle materials. For ductile materials, the choice of theory depends on the level of accuracy required and the degree of computational difficulty the designer is ready to face. For ductile materials, the most accurate way to design is to use distortion energy theory of failure and easiest way to design is to apply maximum shear stress theory.

EXAMPLES

1. A cantilever beam of rectangular cross-section is used to support a pulley as shown, in Fig.7(a). The tension in the wire rope is 5 kN. The beam is made of cast iron FG 200 and the factor of safety is 2.5. The ratio of depth to width of the cross-section is 2. Determine the dimensions of the cross-section of the beam.

Solution. The beam is subjected to a bending moment and the allowable bending stress is given by,

$$\sigma_t = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 \text{ N/mm}^2$$

The forces acting on the beam are shown Fig. 7 (b). Referring to the figure,

$$(M_b)\text{at B} = 5000 \times 500$$
$$= 2500 \times 10^3 \text{ N-mm}$$

And (M$_b$) at A = 5000 x 500 + 5000 x 1500 = 10000 x 10^3 N-mm

The bending moment diagram is shown in Fig. 7 (c) and the cross-section at A is subjected to maximum bending stress. For this cross-section,

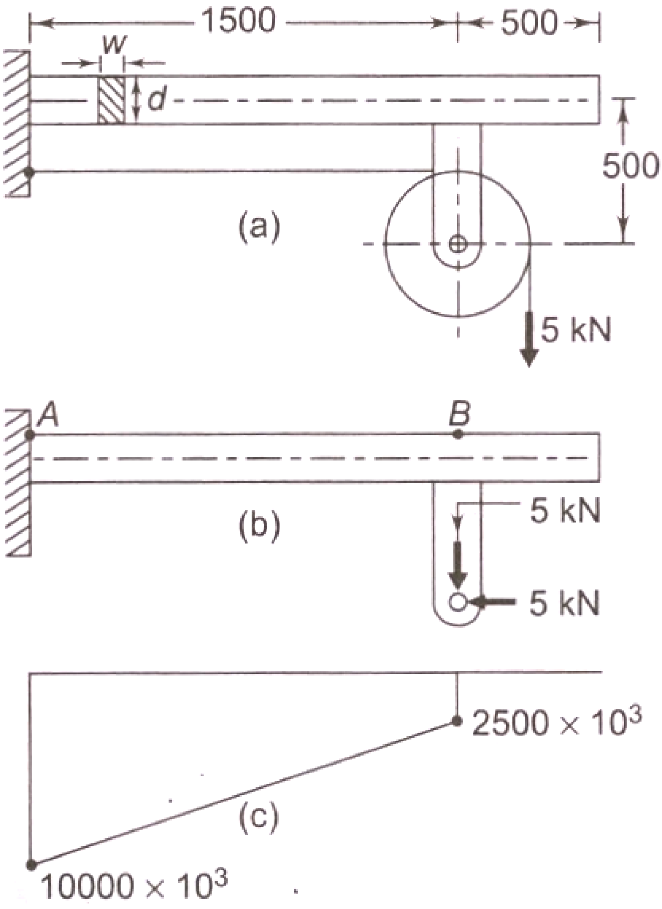


Fig.7

$$d = 2w \quad I = \frac{1}{12}\left[(w)(2w)^3\right] = \frac{2}{3}w^4 \text{mm}^4$$

and $$\sigma_b = \frac{M_b\, y}{I}$$ or $$80 = \frac{(10000 \times 10^3)(w)}{\left(\frac{2}{3}w^4\right)}$$

Therefore, w = 57.24 mm or 60 mm; d = 2w = l20 mm

2. A wall bracket with a rectangular cross-section is shown in Fig.8. The depth of the cross-section is twice of the width. The force P acting on the bracket at 60° to the vertical is 5 kN. The material of the bracket is grey cast-iron PG 200 and the factor of safety is 3.5. Determine the dimensions of the cross-section of the bracket. Assume maximum principal stress theory of failure.

Solution. The stress is maximum at point A in section XX. The point is subjected to combined bending and direct tensile stresses. The force P is resolved into two components: horizontal component P_h and vertical component $P_{v.}$

P_h = P sin60° = 5000 sin60° = 4330.13 N

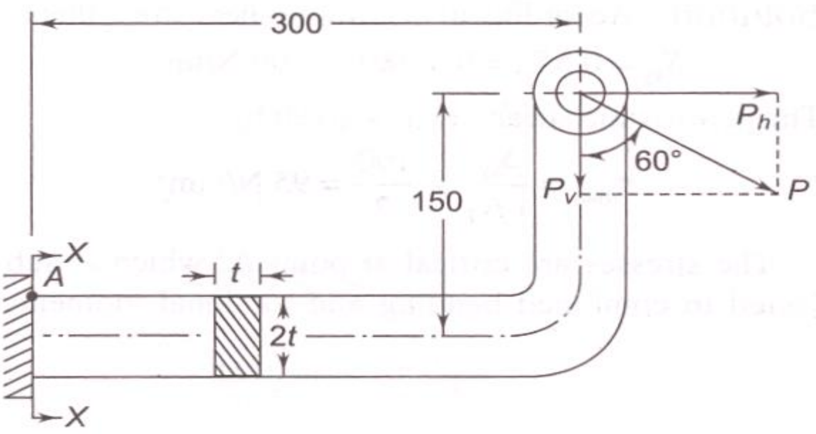
P_v = P cos60° = 5000 cos6o° = 2500 N

The bending moment at section XX is given by,

$$M_b = P_h \times 150 + P_v \times 300$$
$$= 4330.13 \times 150 + 2500 \times 300$$
$$= 1399.52 \times 10^3 \text{ N-mm}$$

$$\sigma_b = \frac{M_b\, y}{I} = \frac{(1399.52 \times 10^3)(t)}{\left[\frac{1}{12}(t)(2t)^3\right]}$$
$$= \frac{2099.28 \times 10^3}{t^3} \text{ N/mm}^2$$

Fig. 8. Wall bracket



The direct tensile stress due to component h is given by,

$$\sigma_t = \frac{P_h}{A} = \frac{4330.13}{2t^2} = \frac{2165.07}{t^2} \text{ N/mm}^2$$

The vertical component P induces shear stress at section XX. It is however, small and neglected. The resultant tensile stress σ_{max} at point A is given by,

$$\sigma_{max} = \sigma_b + \sigma_t = \frac{2099.28 \times 10^3}{t^3} + \frac{2165.07}{t^2} \qquad \text{(i)}$$

The permissible tensile stress is given by,

$$\sigma_{max} = \frac{S_{ut}}{(fs)} = \frac{200}{3.5} = 57.14 \text{ N/mm}^2 \qquad \text{(ii)}$$

Equating (i) and (ii),

$$\frac{2099.28 \times 10^3}{t^3} + \frac{2165.07}{t^2} = 57.14$$

or

$$t^3 - 37.89\, t - 36739.24 = 0$$

Solving the above cubic equation by trial and error method,

t = 33.65 mm ≅ 35 mm

The dimensions of the cross-section are 35 x 70 mm.

1. The shaft of an overhang crank subjected to a force P of 1 kN is shown in Fig. 9. The shaft is made of plain carbon steel 45C8 and the tensile yield strength is 380 N/mm^2. The factor of safety is 2. Determine the diameter of the shaft using the maximum shear stress theory.

Solution. According to maximum shear stress theory,

$$S_{sy} = 0.5 S_{yt} = 0.5(380) = 190 \text{ N/mm}^2$$

The permissible shear stress is given by,

$$\tau_{max} = \frac{S_{sy}}{(fs)} = \frac{190}{2} = 95 \text{ N/mm}^2 \qquad \text{(i)}$$

The stresses are critical at point A, which is subjected to combined bending and torsional moments.

At point A,

$$M_b = P \times (250) = (1000)(250) = 250 \times 10^3 \text{ N-mm}$$
$$M_t = P \times (500) = (1000)(500) = 500 \times 10^3 \text{ N-mm}$$

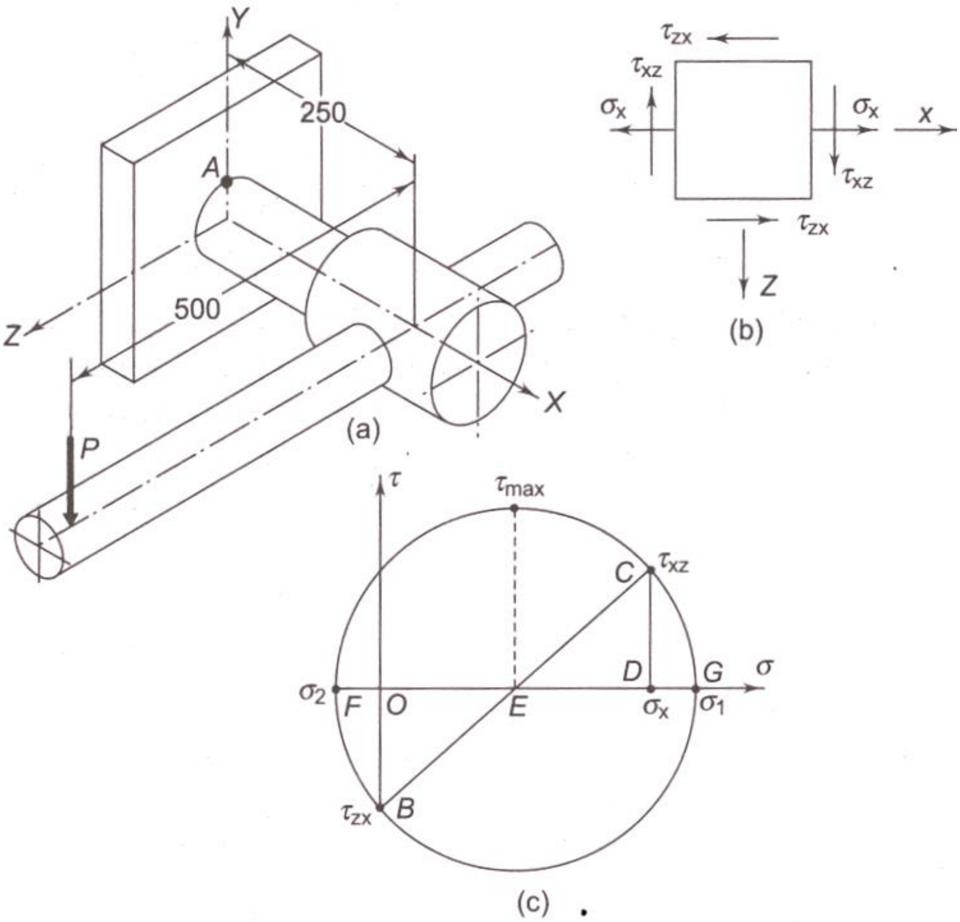


Fig.9

$$\sigma_b = \frac{M_b\, y}{I} = \frac{(250 \times 10^3)(d/2)}{(\pi d^4/64)}$$

$$= \left(\frac{2546.48 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\tau = \frac{M_t\, r}{J} = \frac{(500 \times 10^3)(d/2)}{(\pi d^4/32)}$$

$$= \left(\frac{2546.48 \times 10^3}{d^3}\right) \text{N/mm}^2$$

The stresses at point A and corresponding Mohr's circle are shown in Fig. 9 (b) and (c) respectively. In these figures,

$$\sigma_x = \sigma_b = \left(\frac{2546.48 \times 10^3}{d^3}\right) N/mm^2 \qquad where\ \sigma_y = 0$$

$$\tau = \quad \tau_{xz} = \tau_{zx} = \left(\frac{2546.48 \times 10^3}{d^3}\right) N/mm^2$$

Mohr's circle,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2}$$

$$= \left[\sqrt{\left(\frac{2546.48}{2\,d^3}\right)^2 + \left(\frac{2546.48}{d^3}\right)^2}\,\right] \times 10^3$$

$$= \frac{2847.05 \times 10^3}{d^3} \qquad \text{(ii)}$$

Equating (i) and (ii),

$$\frac{2847.05 \times 10^3}{d^3} = 95 \qquad \therefore d = 31.06 \text{ mm}$$

2. The dimensions of an overhang crank are given in Fig. 10. The force P acting at the crank pin is 1 kN. The crank is made of steel 30C8 (Syt = 400 N/mm^2) and the factor of safety is 2. Using maximum shear stress theory of failure, determine the diameter d at section XX.

Solution: According to maximum shear stress theory,

$$S_{sy} = 0.5S_{yt} = 0.5(400) = 200 \text{ N/ mm}^2$$

The permissible shear stress is given by,

$$\tau_{max} = \frac{S_{sy}}{(fs)} = \frac{200}{2} = 100 \text{ N/mm}^2 \qquad \text{(i)}$$

The section of the crankpin at section XX is subjected to combined bending and torsional moments. At section XX,

$$M_b = 1000 \times (50 + 25 + 100) = 175 \times 10^3 \text{ N-mm}$$
$$M_t = 1000 \times 500 = 500 \times 10^3 \text{ N-mm}$$

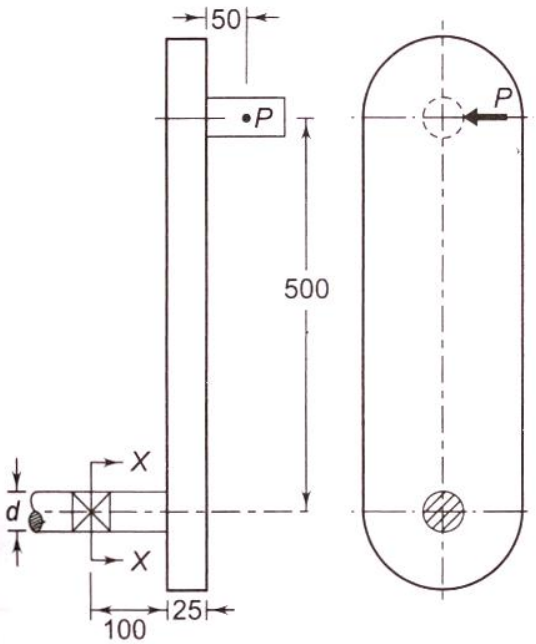


Fig.10. Overhang crank

$$\sigma_x = \sigma_b = \frac{M_b y}{I} = \frac{(175 \times 10^3)(d/2)}{(\pi d^4/64)}$$

$$= \left(\frac{1782.54 \times 10^3}{d^3}\right) \text{N/mm}^2$$

$$\sigma_y = 0$$

$$\tau = \frac{M_t r}{J} = \frac{(500 \times 10^3)(d/2)}{(\pi d^4/32)}$$

$$= \left(\frac{2546.48 \times 10^3}{d^3}\right)$$

The problem is similar to the previous one and the maximum shear stress is given by, [Figs 9(b) and (c)]

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{1782.54 \times 10^3}{2\, d^3}\right)^2 + \left(\frac{2546.48 \times 10^3}{d^3}\right)^2}$$

$$= \frac{2697.95 \times 10^3}{d^3} \text{N/mm}^2 \qquad \text{(ii)}$$

Equating (I) and (ii),

$$\frac{2697.95 \times 10^3}{d^3} = 100 \qquad \therefore d = 29.99 \text{ or } 30 \text{ mm}$$

EXERCISES

1. A C-frame subjected to a force of 15 kN is shown in Fig. 11. It is made of grey cast iron FG 300 and the factor of safety is 2.5. Determine the dimensions of the cross-section of the frame.

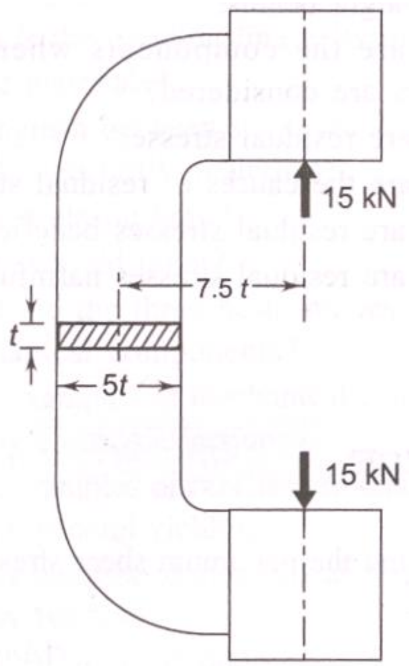


Fig. 11

2. The principal stresses induced at a point in a machine component made of steel 50C4 (S_{yt} = 460 N/mm^2) are as follows:

σ_1 = 200N/mm2 σ_2 =150N/mm2 σ_3 =0

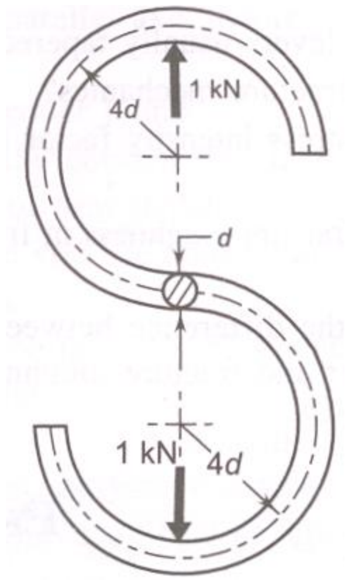
Calculate the factor of safety by (i) the maximum shear stress theory and (ii) the distortion energy theory.

3. The stresses induced at a critical point in a machine component made of steel 45C8 (S_{yt} = 380 N/mm^2) are as follows:

σ_x = 100 N/mm^2 σ_y = 40 N/mm τ_{xy} = 80 N/mm^2

Calculate the factor of safety by (i) the maximum normal stress theory, (ii) the maximum shear stress theory and (iii) the distortion energy theory.

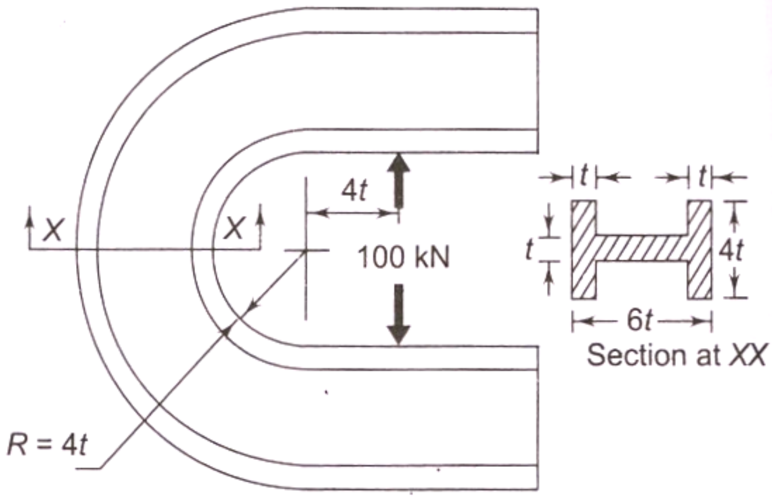
4. A link of S-shape made of a round steel bar is shown in Fig.12. It is made of plain carbon steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 4.5. Calculate the dimensions of the link.



Fig,12

5. The frame of a 100 kN capacity press is in Fig.13. It is made of grey cast iron and the factor of safety is 2.5. Determine dimensions of the cross-section at xx.

Fig.13



6. A bracket, made of steel FeE 200 (S_{yt} = 200 N/mm^2) and subjected to a force of 5 kN acting at an angle of 30^o to the vertical, is shown in Fig. 14.

Fig.14